Приложение 1.2
УТВЕРЖДЕНО
приказом МФТИ
от 11.12.2019 г. № 2094-1

Appendix 1.2
APPROVED
by MIPT order № 2094-1
December 11, 2019

**Entrance Examination in Mathematics**
**for students to be readmitted after expulsion or be transferred from other universities to MIPT**

**Test Format**

The entrance test shall take the form of a combination of a written and oral examination.

1. The exam takes place in accordance with the schedule established by MIPT order.

2. Only students having a valid Exam Sheet issued by the Admissions Office are allowed to sit a test.

3. The total written test time is 2 hours and the test is held in the classroom specially designated.

4. Test takers are required to fill in a specific exam form given by examiners.

5. It is absolutely forbidden to use any kind of study materials, means of communication, and any electronic devices (including calculators). In case of violation test takers are expelled from the classroom and "unsatisfactory" mark is given in the Academic Transcript.

6. The written test card for test takers in "Aerospace Engineering" includes problems on the topics listed below:

- for semester 2: sections I, II
- for semester 3: sections I, II, III, IV
- for semester 4: sections I, II, III, IV, V, VI
- for semester 5: sections I, II, III, IV, V, VI, VII, VIII
- for semester 6: sections I, II, III, IV, V, VI, VII, VIII, IX, XII
- for semester 7 through semester 8: sections I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII

7. The written test card for test takers in "Biomedical Engineering" includes problems on the topics listed below:

- for semester 2: sections I, II
- for semester 3: sections I, II, III, IV
- for semester 4: sections I, II, III, IV, V, VI
- for semester 5: sections I, II, III, IV, V, VI, VII, VIII
- for semester 6: sections I, II, III, IV, V, VI, VII, VIII, IX
- for semester 7 through semester 8: sections I, II, III, IV, V, VI, VII, VIII, IX, XI

8. The written test card for test takers in "Computer Science" includes problems on the topics listed below:

- for semester 2: sections I, II
- for semester 3: sections I, II, III, IV
- for semester 4: sections I, II, III, IV, V, VI
- for semester 5: sections I, II, III, IV, V, VI, VII, VIII

- for semester 6 through semester 8: sections I, II, III, IV, V, VI, VII, VIII, IX, X

9. After the written test being completed and checked, face-to-face discussion of the written part is held with each applicant.

10. Examiners have the right to ask supplementary questions to test takers in "Aerospace Engineering" on the topics listed below:

- for semester 2: sections I, II
- for semester 3: sections I, II, III, IV
- for semester 4: sections I, II, III, IV, V, VI
- for semester 5: sections I, II, III, IV, V, VI, VII, VIII
- for semester 6: sections I, II, III, IV, V, VI, VII, VIII, IX, XII
- for semester 7 through semester 8: sections I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII

11. Examiners have the right to ask supplementary questions to test takers in "Biomedical Engineering" on the topics listed below:

- for semester 2: sections I, II
- for semester 3: sections I, II, III, IV
- for semester 4: sections I, II, III, IV, V, VI
- for semester 5: sections I, II, III, IV, V, VI, VII, VIII
- for semester 6: sections I, II, III, IV, V, VI, VII, VIII, IX
- for semester 7 through semester 8: sections I, II, III, IV, V, VI, VII, VIII, IX, XI

12. Examiners have the right to ask supplementary questions to test takers in "Computer Science" on the topics listed below:

- for semester 2: sections I, II
- for semester 3: sections I, II, III, IV
- for semester 4: sections I, II, III, IV, V, VI
- for semester 5: sections I, II, III, IV, V, VI, VII, VIII
- for semester 6 through semester 8: sections I, II, III, IV, V, VI, VII, VIII, IX, X

13. Test results are reported as a band score on a ten-point scale, with 0-2 being unsatisfactory, 3-4 being satisfactory, 5-7 being good, and with 8-10 being excellent.

**Entrance Examination in Mathematics**

## I. Introduction to calculus

1. Linear equations. Quadratic equations. Rational equations. Module equations. Equations of higher degrees. Factorization of polynomials. Absolute-value inequalities.
2. Root of a number and its properties. Arithmetic root. Irrational equations. Irrational inequalities.
3. Arithmetic and geometric progressions and their properties.
4. Trigonometric formulas. Trigonometric and inverse trigonometric functions and their properties. Conversion of trigonometric expressions. Trigonometric equations and inequalities.
5. Real numbers. Bounded (from above or below) numerical sets. The axiom of completeness and the existence of a least upper (greatest lower) bound of a set. Unboundedness of the set of natural numbers. Fundamental lemmas connected with the completeness of the set of real numbers R (nested interval lemma, finite covering, limit point).
6. Complex numbers. Modulus and argument. Trigonometric form. Arithmetic operations with complex numbers. Euler's formula.
7. The limit of a numerical sequence. The uniqueness of a limit. Infinitesimal sequences and their properties. Properties of limits related to inequalities. Arithmetic operations with convergent sequences. Monotone Convergence Theorem. The number $e$. The nested interval lemma (Cauchy-Cantor principle). Infinitely large sequences and their properties.
8. Subsequences, partial limits. Upper and lower limits of the numerical sequence. Bolzano–Weierstrass theorem. Cauchy Convergence Criterion.
9. Limit of a function of one variable. Heine and Cauchy definitions, their equivalence. Properties of the limit of a function. Different types of limits. The Cauchy criterion for the existence of a limit of a function. The limit of a composite function. The existence of one-sided limits for a monotonic function.
10. Continuity of a function at a point. Properties of continuous functions. One-sided continuity. Continuity of a composite function. Points of discontinuity, their classification. Points of discontinuity of monotonic functions.
11. Properties of continuous functions on closed intervals — boundedness, attainment of the supremum and the infimum. Intermediate value theorem. Inverse function theorem.
12. Continuity of elementary functions. The definition of the exponential function. Properties of the exponential function. Limits $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ and $\lim\limits_{x\to 0}\frac{\sin x}{x}$ and corollaries.
13. Comparison of values (symbols $o$, $O$, $\sim$). Replacement of functions by other functions equivalent to them in a given base when computing limits of monomials
14. Derivative of a function of one variable. One-sided derivatives. The continuity of a function having a derivative. Functions differentiate at a point, differential. Geometric meaning of the derivative and differential. Differentiation and the Arithmetic Operations. Derivative of a composite function. Derivative of an inverse function. Derivatives of elementary functions. Invariance of a form of a differential with respect to the change of variable.

## II. Geometry

1. Points, lines, line segments. Ray. Angle. Comparison of segments and angles. Measurement of segments. Measurement of angles. Adjacent and vertical angles. Perpendicular lines. Parallel lines.
2. SSS, SAS, ASA and AAS rules. Medians, bisectors and altitudes of a triangle. Properties of an isosceles triangle. Sum of the angles of a triangle. Equilateral triangles. Area of a triangle. Pythagorean Theorem. Similar triangles. Law of sines. Law of cosines.
3. Convexpolygon. Quadrilateral. Parallelogram. Characteristics of a parallelogram. Trapezoid. Rectangle. Rhombus. Square.
4. Tangent to a circle. Degree measure of an arc of a circle. Inscribed angle theorem. Inscribed circle. Circumscribed circle. Length of a circle and area of a circle.
5. Polyhedron. Parallelepiped. Prism. Cylinder. Cone. Sphere and ball. Bodyvolume.

6. Addition of matrices and multiplication by numbers. Matrix multiplication and inversion. Determinants of square matrices of 2-nd and 3-rd orders. Solving systems of linear equations by the Cramer method.
7. Linear spaces and their basic properties. Vectors and operations with them. Commutativity, associativity and distributivity of vector operations.
8. Linearly dependent and linearly independent systems of vectors. Basis, coordinates of vectors in the basis. Coordinate representation of vectors. Operations with vectors in coordinate representation. Change of coordinates basis. Necessary and sufficient condition for linear dependence of vectors in the coordinate form.
9. Change of coordinate system. Transition matrix and its properties.
10. Orthogonal projections of vectors and their properties. Dot product: properties, coordinate expression. Formulas for the distance between two points and the angle between two directions.
11. Oriented set of vectors. Cross product, its properties, expression in orthonormal basis. Geometric meaning of a vector product. Expression of a cross product in an arbitrary basis.
12. Triple product of vectors, its properties, expression in arbitrary and orthonormal bases. The geometric meaning of the triple product. Conditions of collinearity and coplanarity of vectors. The formula of a double vector product.
13. Line on a plane. Vector and coordinate equations of a line. Positional and metric problems on lines on a plane. Translation of one form of description of lines on a plane into other form.

## III. Single Variable Calculus

1. Higher-order derivatives. Leibniz' formula for the $n$-th derivative of a product. Differential of the second order. Higher-order differentials.
2. Local extrema of a function. A necessary condition for an interior extremum of a differentiable function (Fermat's lemma). Rolle's Theorem. The finite-increment theorems of Lagrange and Cauchy (mean-value theorems). Taylor's formula with the Peano and Lagrange forms of the remainder. L'Hospital's rule for $\frac{0}{0}$ and $\frac{\infty}{\infty}$ indeterminate forms.
3. Differential calculus used to study functions. Necessary and sufficient conditions for a function to be monotonic, necessary conditions for an interior extremum of a function in term of the first derivative. Sufficient conditions for an extremum in terms of higherorder derivatives. Conditions for a function to be convex. Inflection points. Constructing the Graph of a Function - asymptotes, finding intervals of monotonicity of the function and its local extreme values, intervals of convexity and the points of inflection.
4. Primitive and indefinite integral. Linearity of the indefinite integral, integration by substitution and by parts. Integration of rational functions. Basic techniques for integrating irrational and transcendental functions.
5. Curves on a plane and in the space. Smooth curves, tangent to a smooth curve. Mean value theorem for vector-valued functions. The length of a curve. Arc length derivation. Natural parameter. Curvature of a curve, formulas for its calculation.
6. Jordan measure in n-dimensional Euclidean space. Measurement criterion. The measurability of the union, intersection, and difference of measurable sets. The finite additivity of the Jordan measure.
7. The Riemann integral on a closed interval. Riemann sums, Darboux sums, a criterion for integrability. Integrability of a continious function, integrability of a monotonic function, integrability of a bounded function with finite number of points of discontinious. Properties of integrable functions: additivity of the integral of intervals of integration, linearity of an integral, integrability of a product of functions, integrability of an absolute value of the integrable function, integration of inequalities, mean-value theorem. Properties of an integral with variable upper limit — continuity, differentiability. The Newton-Leibniz Formula. Integration by parts and by substitution.
8. Geometric applications of a definite integral — the area of a curvilinear trapezoid, the volume of a body of revolution, the length of a curve, the surface area of a revolution.
9. Curvilinear integral of the first kind and its properties. Orientation of a smooth curve. Curvilinear integral of the second kind and its properties.
10. Improper Integrals (cases of unbounded function and infinite interval). Cauchy criterion for convergence of an improper integral. Integrals of functions of constant signs. Theorems for studying the convergence of an improper integral. Integrals from alternating functions: convergence and absolute convergence. Dirichlet's and Abel's tests for the convergence of integrals.

## IV. Analytic geometry

1. Straight lines in the space. Vector and coordinate representations of a straight line in the space. Plane in the space. Representations of a plane in the space. Positional and metric problems on lines and planes in space. Transforming between different line and plane forms. Line bundle. Bundle of planes. Linear inequalities.
2. Ellipse, hyperbola and parabola, their properties. Tangent to ellipse, hyperbola and parabola. Central lines.
3. Ellipsoids, hyperboloids and paraboloids. Main properties. Rectilinear generators. Cylinders and cones. Surfaces of revolution. Classification and canonical equations of second-order algebraic surfaces.
4. Mapping and transformations of a plane. Composition of mappings. One-to-one (bijective) mapping. Inverse mapping. Linear transformations of a plane and their properties. Coordinate representation of linear transformations of a plane.
5. Affine transformations and their properties. The main directions of affine transformations. The geometric meaning of the modulus and sign of the determinant of an affine transformation matrix.
6. Multiplication and inversion of matrices. Orthogonal matrices. Elementary matrix operations. The matrix form of elementary transformations.
7. Definition and basic properties of determinants of matrices. Minors, expansion of a determinant along row or column. Expansion using minors and cofactors. Determinant of matrix product.
8. Inverse matrix. Invertible matrix theorem. Explicit formula for inverse matrix elements.
9. Rank of a matrix. Basis minor theorem. Matrix rank theorem.
10. Systems of linear equations. Kronecker – Capelli theorem. Fundamental system of solutions and the general solution of a homogeneous system of linear equations. General solution to a nonhomogeneous system. Gauss method. Fredholm theorem.

## V. Multivariable Calculus

1. Points $n$-dimensional space. Distance between points, its properties. Limit of a sequence of points in $n$-dimensional Euclidean space. The Bolzano – Weierstrass theorem and the Cauchy criterion for convergence of a sequence. Interior, limit, isolated points of a set, points of closure. Open and closed sets, their properties. Interior, closure and boundary of a set.
2. Limit of a function of several variable. Definitions in terms of neighborhoods and in terms of sequences. Limit of a function over a base. Directional limits. Iterated limits. Investigation of the limit of a function of two variables changing to polar coordinates.
3. Continuity of a function of several variables. Continuity over a base. Continuity of a composite function. Properties of continuous functions on a compact set — boundedness, attainability of the infimum and supremum, and uniform continuity. Intermediate value theorem.
4. Partial derivatives of functions of several variables. Differentiability of a function of several variables at a point, differential. Necessary conditions for differentiability, sufficient conditions for differentiability. Differentiability of a composite function. Invariance of the differential form with respect to the change of variables. Gradient, its independence from the choice of a rectangular coordinate system. Directional derivative.
5. Partial derivatives of higher orders. Independence of the mixed partial derivative of the differentiation order. Differentials of higher orders. Taylor's formula for functions of several variables with the remainder term in Lanrange and Peano form.
6. Numerical series. Cauchy's convergence test. Criterion for convergence of series of nonnegative terms: comparison theorem, d'Alembert and Cauchy tests, integral test. Alternating series: convergence and absolute convergence. Abel-Dirichlet test for convergence of an integral.
7. Uniform convergence of functional sequences and series. Cauchy criterion for uniform convergence. Weierstrass test for uniform convergence of functional series. Continuity of the sum of a uniform convergent series of continuous functions. Term by term integration and differentiation of functional sequences and series. Dirichlet and Abel tests.
8. Power series with complex terms. Abel's first theorem. Disk and radius of convergence. Character of convergence of a power series in a disk of convergence. The Cauchy – Hadamard formula for the radius of convergence. Continuity of the sum of a complex power series.

9. Power series with real terms. Preservation of a radius of convergence of a power series in term-by-term differentiation and integration. Infinite differentiability of the sum of a power series on the interval of convergence. Uniqueness of power series representation of a function, Taylor series. Taylor formula with the remainder term in integral form. An example of an infinitely differentiable function that has not power series representation. Taylor representation of basic elementary functions. Power series expansion of the complex-valued function $e^z$.
10. The implicit function theorem. Extrema of functions of several variables (necessary and sufficient conditions for an interior extremum). Conditional extremum. Method of Lagrange multipliers. Necessary conditions. Sufficient conditions.
11. Multiple integrals. Riemann and Darboux sums. Tests for integrability. Integrability of a continious function on a measurable compact set. Properties of integrable functions. Reduction of a multiple integral to an iterated integral. Jacobian matrix and determinant. Change of variables in multiple integrals.
12. Green's theorem. Potential vector fields on a plane. Path independence of line integrals.
13. Simple smooth surfaces. Surface integrals of first and second kinds. Area of a surface. Orientation of a simple smooth surface. Piecewise smooth surfaces, their orientation and integrals over them.
14. The Gauss-Ostrogradskii Formula. Divergence of a vector field, its geometrical meaning. Solenoidal vector fields. Stokes' Formula. Potential vector fields. Nabla operator and operations with it.

## VI. Linear algebra

1. Vector spaces. Linear dependence and linear independence of vectors in linear spaces. Basis and dimension.
2. Basis expansion in vector spaces. Coordinate representation of vectors and operations with it. Isomorphism theorem. Change of basis. Transition matrix.
3. Subspaces and spanning sets. Sum and intersection of subspaces. Direct sum. Dimension of a sum of subspaces.
4. Linear transformations and linear mappings. Kernel and image of a linear mapping. Inverse mapping.
5. Matrices of linear mappings and transformations for finite-dimensional spaces. Algebraic operations with mappings in the coordinate form. Changing of a matrix of linear mapping when we change bases. Isomorphism of the space of linear mappings and the space of matrices.
6. Invariant subspaces of linear transformations. Eigenvectors and eigenvalues. Eigenspaces. Linear independence of eigenvectors corresponding to different eigenvalues.
7. Finding eigenvectors and eigenvalues of linear transformation. Characteristic equation. Invariance of characteristic equation. Estimation of the dimension of eigenspace. Diagonalizable matrices of linear mappings.
8. Bilinear and quadratic forms. Their coordinate representation in finite-dimensional vector space. Change of matrices of bilinear and quadratic forms when we change a basis.
9. Lagrange reduction of quadratic form to canonical form. Sylvester's law of inertia. Definite quadratic forms. Sylvester's criterion.
10. Reduction of a quadratic form to a diagonal form by elementary transformations. Axiomatics of Euclidean space. Cauchy–Bunyakovsky–Schwarz inequality. Triangle inequality. Gram matrix and its properties.
11. Finite-dimensional Euclidean space. Gram–Schmidt process. Change of orthonormal basis. Orthogonal complement of subspace.
12. Linear transformations of Euclidean space. Orthogonal projection into a subspace. Adjoint transformations, their properties. Matrix of adjoint transformation.
13. Self-adjoint transformations. Properties of their eigenvectors and eigenvalues. Existence of an orthonormal basis of eigenvectors of a self-adjoint transformation.
14. Orthogonal transformations. Their properties. Orthogonal matrices.
15. The construction of an orthonormal basis in which the quadratic form has a diagonal form. Simultaneous reduction to a diagonal form of a pair of quadratic forms, one of which is positive-definite.

## VII. Fourier Analysis

1. The Riemann-Lebesgue Lemma. Trigonometric Fourier series for absolutely integrable functions, the tendency of their coefficients to zero. Representation of the partial sum of the Fourier series by an integral

through the Dirichlet kernel. Localization principle. Sufficient conditions for a Fourier series to Converge at a point. Uniform convergence of Fourier series. Term-by-term differentiation and integration of Fourier series. Smoothness of a function and the rate of convergence of its Fourier series. Fourier series in complex form.

2. Summation of Fourier series by the arithmetic mean method. Weierstrass' theorem on approximation by trigonometric and algebraic polynomials

3. Metric and normed vector spaces. Convergence in metric spaces. Complete metric spaces, complete normed vector spaces. Completeness of the space $C[a;b]$. Comparison of norms: comparison of uniform convergence, convergence in mean and quadratic mean. Complete systems in normed linear spaces.

4. Infinite-dimensional Euclidean spaces. Fourier series in an orthonormal system. Minimal property of Fourier coefficients, Bessel inequality. Parseval equality. Orthonormal basis in infinite-dimensional Euclidean space. Hilbert spaces. A necessary and sufficient condition for a sequence of numbers to be a sequence of Fourier coefficients of an element of a Hilbert space with a fixed orthonormal basis. The connection between the concepts of completeness and closedness of an orthonormal system.

5. Trigonometric Fourier series for functions which are absolutely square integrable. Completeness of the trigonometric system, Parseval equality. Completeness of the Legendre polynomial system.

6. Proper integrals depending on a parameter, their properties. Cauchy criterion and the basic sufficient conditions for convergence (majorant, Abel-Dirichlet). Improper integrals depending on a parameter, uniform convergence. Cauchy criterion and the basic sufficient conditions for uniform convergence (M-test, Abel-Dirichlet). Continuity, differentiation, and integration of an improper integral depending on a parameter. Dirichlet and Laplas integral.

7. Fourier integral. Representation of a function by means of a Fourier integral. The Fourier transform of an absolutely integrable function and its properties: continuity, tending to zero at infinity. Fourier inversion theorem. Fourier transform of the derivative and the derivative of the Fourier transform.

## VIII. Differential equations 1

1. The simplest types of the first order differential equations: separable equations, homogeneous equations, linear equations, exact differential equations. Integrating factor. Bernoulli or Riccati equation. The method of introducing a parameter for first order implicit differential equation. Reduction of order of differential equations.

2. Linear differential equations and linear systems of differential equations with constant coefficients. The formula for the general solution of $n$-th order linear homogeneous equation. Finding a solution to a linear inhomogeneous equation in the case when the right-hand side of the equation is a quasi-polynomial. Euler equation.

3. The formula for the general solution of a linear homogeneous system of equations in the case of simple eigenvalues of the matrix of coefficients of the system. The formula for the general solution of a linear homogeneous system in the case of repeated eigenvalues of the coefficient matrix of the system. Finding a solution to a linear inhomogeneous system of equations in the case when the free terms of the equation are quasi-polynomials.

4. Matrix exponential and its use to obtain formulas for the general solution and the solution of the Cauchy problem for linear homogeneous and inhomogeneous systems of equations.

## IX. Differential equations 2

1. The simplest problem of calculus of variations. The problem with free ends. Necessary condition for a weak local extreme, Euler equation.

2. Cauchy problem. Existence and uniqueness of solutions to the Cauchy problem for a normal system of differential equations and for an $n$-th order equation in the normal form. Singular Solution.

3. Autonomous systems of differential equations. Basic concepts and properties of phase trajectories. Classification of equilibrium points of linear autonomous systems of second-order equations. Behavior of phase trajectories in the neighborhood of the equilibrium points of two-dimensional autonomous nonlinear systems of equations.

4. First integrals and the first order linear homogeneous partial differential equations. First integrals of systems of ordinary differential equations. Criterion for the first integral. Theorem on the number of independent first integrals. The formula for the general solution of a first order linear homogeneous partial differential equation. Statement of the Cauchy problem for such equations. Existence and uniqueness theorem to the solution of the Cauchy problem.
5. Linear differential equations and linear systems of differential equations with variable coefficients. Existence and uniqueness of the solution to the Cauchy problem for normal linear systems of equations and for $n$-th order linear equation in the normal form. Fundamental system and fundamental matrix of solutions to a linear homogeneous system of equations. The structure of the general solution to a linear homogeneous and heterogeneous system of equations. Wronskian. Liouville-Ostrogradski formula. Variation of constants or the Cauchy formula for a linear inhomogeneous system of equations. Corollaries to $n$-th order linear equations.
6. Sturm's theorem and its corollaries. Bessel equation and some properties of its solutions.

## X. Functions of One Complex Variable

1. Complex numbers. Extended complex plane. Riemann sphere. Sequences and series. Functions of a Complex Variable. Continuous Functions.
2. Differentiation by a complex variable. Cauchy-Riemann conditions. Regular functions. Conjugate harmonic functions of two variables.
3. Elementary functions of a complex variable: power, rational, exponential and trigonometric, their properties. Inverse function theorem. The concept of a multi-valued function and its regular branches. Main regular branches of multi-valued functions $^n\sqrt{z}$ and Ln$z$.
4. Complex integrals. Properties of the integral of a continuous curve along a piecewise smooth contour. Cauchy's integral theorem for regular functions.
5. Cauchy's integral formula. Cauchy-Type Integral, its regularity.
6. Primitive. A sufficient condition for the existence of a primitive. Newton–Leibniz formula.
7. Power series. Abel's first theorem. Radius and circle of convergence of a power series. Expansion in a power series of a function regular in a circle. Uniqueness theorem for regular functions.
8. Power series. Abel's first theorem. Radius and circle of convergence of a power series. Expansion in a power series of a regular function in a circle. Uniqueness theorem for regular functions.
9. Weierstrass theorems for uniformly converging series of regular functions and corollaries to them.
10. Laurent series and its ring of convergence. Laurent expansion of a regular function in a ring, its uniqueness and Cauchy inequality for the coefficients of a Laurent series.
11. Isolated singular points of an unambiguous kind, their classification. Determining the kind of a singular point from the main part of the Laurent series.
12. Residues. Cauchy's residue theorem. Formula for calculating residue. Jordan's lemma. Entire functions and their properties.
13. Argument principle. Rouche's theorem. Fundamental theorem of algebra. Open mapping theorem
14. Geometric meaning of absolute value and argument of derivative. Criterion for conformity of a map at a finite point. Conformal mapping in the extended complex plane.
15. Fractional linear functions and their properties.
16. Conformal mappings by elementary functions. Zhukovsky's function and its properties.

## XI. Partial differential equations

1. Classification of second order linear partial differential equations. Characteristics.
2. Wave equation in the case of one spatial variable. Statement of the Cauchy problem, d'Alembert formula. Domain of dependence of the solution to the Cauchy problem. Continuous dependence of the solution on the initial functions. Example of the absence of continuous dependence in the case of the Laplace equation (Hadamard example).
3. Wave equation in the case of two and three spatial variables. Flat characteristics of the wave equation, light cone. Statement of the Cauchy problem; uniqueness of the solution. Existence of a solution to the Cauchy problem for the wave equation in the case of three spatial variables, Kirchhoff formula. Existence of

a solution to the Cauchy problem for the wave equation in the case of two spatial variables (Poisson formula, gradient descent). Wave propagation in the case of two and three spatial variables. Continuous dependence of a solution on initial functions.
4. Cauchy problem for the heat equation. Fundamental solution of the heat equation. Uniqueness classes of solutions. Existence of a solution, Poisson formula. Infinite differentiability of a solution. Continuous dependence for a solution on an initial function. Maximum principle for the heat equation.
5. Mixed problem for the wave equation and for the heat equation in the case of one spatial variable. Uniqueness of the solution (the method of the energy integral in the case of the wave equation; the maximum principle in the case of the heat equation). Necessary conditions for the solvability of the problem (conditions for matching initial and boundary functions). Fourier method for solving a mixed problem. Justification of the Fourier method.
6. Harmonic functions. Fundamental solution of the Laplace equation. Potentials. Green Formulas. Infinite differentiability of harmonic functions. Average theorems. Maximum principle. Liouville's theorem. Riemann's theorem on removable singularities.
7. Dirichlet problem for the Poisson's equation in a bounded domain. Necessary conditions for solvability. Uniqueness of the solution; continuous dependence of the solution on the boundary function. Existence of a solution to the Dirichlet problem for the Poisson's equation in a ball.
8. Neumann problem for the Poisson equation in a bounded domain. A necessary condition for the solvability of the Neumann problem. A theorem on the general form of a solution. Existence of a solution to the Neumann problem for the Poisson equation in a ball.

## XII. Probability theory

1. Discrete probability space and the classical definition of probability.
2. The calculation of probabilities in the discrete case. Addition theorem on probability for *n* events. Conditional probability. The formula for total probability and the Bayes formula. Independence of events. Some classical discrete probabilistic models and related distributions.
3. Random variables and their numerical characteristics. Independence of random variables. Properties of mathematical expectation and variance associated with the concept of independence. Chebyshev's inequality. The law of large numbers. The covariance of two random variables and its relationship with independence.
4. The general concept of probability space. Probability axioms. Examples of probability spaces.
5. A mathematical model of a sequence of independent experiments. Bernoulli test sequence. Bernoulli scheme.
6. Random variable as a measurable function, its distribution. Main types of distributions. Mathematical expectation of a random variable and its basic properties.
7. Cumulative distribution function and its properties. Joint distribution function of multiple random variables. Pearson's chi-squared test. Multidimensional normal distribution.
8. Characteristic function and its properties. Using characteristic functions to study the sums of independent random variables.
9. Method of characteristic functions in proofs of limit theorems. Strong law of large numbers. Central limit theorem for the sum of identically distributed random variables with finite variance. Poisson's theorem.

## Resources

1. George B. Thomas, Maurice D. Weir, Joel Hass, Frank R. Giordano. Thomas's calculus.
2. Vladimir A. Zorich. Mathematical Analysis I.
3. Vladimir A. Zorich. Mathematical Analysis II.
4. Ruslan A. Sharipov. Course of analytical geometry.
5. Jim Hefferon. Linear Algebra.
6. Ruslan A. Sharipov. Course of linear algebra and multidimensional geometry.
7. Gilbert Strang. Linear algebra and its applications.
8. W. Keith Nicholson. Linear Algebra With Applications.

9. William E. Boyce, Richard C. DiPrima. Elementary Differential Equations and boundary value problems.
10. Dmitri P. Bertsekas, John N. Tsitsiklis. Introduction to Probability, 2nd Edition.
11. Joseph K. Blitzstein, Jessica Hwang. Introduction to Probability.
12. G. Cain. Complex analysis.
13. T. Gamelin. Complex analysis.
14. Yehuda Pinchover, Jacob Rubinstein. An introduction to partial differential equations.

**Head of the Department of Higher Mathematics:** **Grigoriy E. Ivanov**