PROGRAM OF ENTRANCE TEST IN MATHEMATICS FOR STUDENTS ENTERING MASTER PROGRAM «BLOCKCHAIN»

The entrance test in mathematics consists of a written exam (duration – 3 hours) and an oral interview. The final grade is set based on the results of the both parts of the test.

1. Limit of a numerical sequence and its properties. The Cauchy criterion. Partial limit, limit superior and limit inferior. The Bolzano-Weierstrass theorem.
2. Limit of a function of one variable and its properties. Cauchy and Heine definitions of limit, their equivalence. The Cauchy criterion.
3. Continuity of a function at a point. Properties of a continuous function on a closed interval: Weierstrass and Bolzano-Cauchy theorems. Inverse function theorem. Uniform continuity, Cantor's theorem.
4. Derivative at a point of a function of one variable and its properties. Derivative of a composite function. Differentiability of a function at a point, differentiable functions. Differentiation of an inverse function.
5. Higher-order derivatives of a function of one variable. The Leibniz formula.
6. Rolle's theorem. The finite-increment theorems of Lagrange and Cauchy (mean-value theorems).
7. Taylor's formula with the Peano and Lagrange forms of the remainder.
8. The connection between the type of monotonicity of a differentiable function and the sign of its derivative. Sufficient conditions for the presence or absence of a local extremum in terms of the first, second, and higher-order derivatives. Convex functions. Differential conditions for convexity.
9. Differentiability of a function of several variables. Necessary conditions and sufficient conditions for differentiability.
10. The implicit function theorem.
11. Local extremum of a function of several variables. Necessary conditions and sufficient conditions of local extremum.
12. Extrema with constraint (necessary condition). The method of Lagrange’s multipliers, necessary and sufficient conditions for a conditional extremum.
13. Definite integral. Darboux criterion for integrability of a function. Integrals with a variable upper limit of integration, their properties: continuity, differentiability. The Newton – Leibniz formula.
14. Improper integrals. Absolute and conditional convergence. The Cauchy criterion, comparison tests and Dirichlet’s test for convergence of improper integrals.
15. Numerical series. Absolute and conditional convergence. The Cauchy's criterion, the comparison theorem, integral test, Leibniz and Dirichlet tests for convergence of numerical series.
16. Functional series. Uniform convergence. The Cauchy criterion, Weierstrass and Dirichlet tests for uniform convergence.
17. Power series. Radius of convergence, Cauchy-Hadamard formula. Taylor series. Decomposition of elementary functions into Taylor’s series.
18. Curvilinear integral. Green’s formula.
19. Surface integral. The Ostrogradsky-Gauss and Stokes formulas.
20. The Riemann-Lebesgue Lemma. Trigonometric Fourier series for absolutely integrable functions, the tendency of their coefficients to zero. Sufficient conditions for a Fourier series to converge at a point. Uniform convergence of Fourier series.
21. The Fourier transform of an absolutely integrable function and its properties. Fourier transform of the derivative and the derivative of the Fourier transform.
22. The Weierstrass approximation theorem. Complete systems in normed spaces.
23. Different types of representations of straight lines and planes. Angles between straight lines and planes. Distance from a point to a line and a plane. Distance between skew lines.
24. Second order curves. Ellipse, parabola, hyperbola and their properties.

25. Affine transformations and their properties. The main directions of affine transformations. The geometric meaning of the absolute value and sign of the determinant of an affine transformation matrix.
26. Orthogonal transformations of a plane and its properties. Decomposition of an affine transformation into an orthogonal transformation and two shearings.
27. Systems of linear algebraic equations. Kramer's rule. Rouché–Capelli theorem. Fredholm's Theorem. General solution to a system of linear equations.
28. Linear transformation of a finite-dimensional space, its matrix. Change of basis. Eigenvectors and eigenvalues, their properties.
29. Quadratic forms and their reduction to the canonical form.
30. Finite-dimensional Euclidean spaces. The Gram Matrix. Conjugate linear transformation of a finite-dimensional Euclidean space and its properties.
31. Self-adjoint linear transformations of a finite-dimensional Euclidean space, properties of its eigenvalues and eigenvectors.
32. Ordinary differential equations. Separation of variables. Reduction of order of differential equations. Introducing a parameter.
33. Linear differential equations and linear systems of differential equations with constant coefficients. Finding solutions.
34. Linear differential equations and linear systems of differential equations with variable coefficients. Fundamental system of solutions. Wronskian. Liouville-Ostrogradski formula. Variation of constants.
35. The simplest problem of calculus of variations. Necessary condition for a weak local extreme, Euler equation.
36. Autonomous systems of differential equations. Classification of equilibrium points of linear autonomous systems of second-order equations. Stability and asymptotic stability of the equilibrium point.
37. First integrals of an autonomous system of differential equations. Theorem on the number of independent first integrals. Linear differential equations in partial derivatives, general solution to the Cauchy problem.
38. Probability space. Independent events. Addition theorem of probability. Conditional probability. A complete system of events. The formula of total probability. Bayes formula.
39. Random variable and its distribution. Mathematical expectation and the variance of the random variable and their properties.
40. Bernoulli scheme. Chebyshev's inequality and the law of large numbers.
41. Regular functions of a complex variable. Cauchy integral formula. Ring of regular functions. Laurent series.
42. Residues. Cauchy's residue theorem. Formula for calculating residue. Jordan's lemma. Entire functions and their properties.
43. Regular branches of multivalued complex functions $\sqrt{zn}$ and Ln(z) and their application for calculating integrals.
44. Conformal mappings. Fractional-linear mapping and its properties. Zhukovsky's function and its properties.
45. Second order linear partial differential equations in two variables that are hyperbolic in a given domain. Method of characteristics for the search of general solution and the solution of Cauchy's problem.
46. Cauchy problem for the wave equation and one-dimensional heat equation. D'Alembert and Poisson formulas.
47. Mixed problem for the wave for a semi-infinite string. Initial and boundary conditions.
48. Cauchy problem for the wave equation in three-dimensional space. Kirchhoff formula.
49. Internal and external Dirichlet and Neumann problems for Laplace and Poisson's equations in a circle and a ball.
50. Fourier method for solving a mixed problem for the wave and heat equations.

51. Fredholm integral equations of the second kind with degenerate kernels.

Literature for self-study

1. George B. Thomas, Maurice D. Weir, Joel Hass, Frank R. Giordano. Thomas's calculus.
2. Vladimir A. Zorich. Mathematical Analysis I.
3. Vladimir A. Zorich. Mathematical Analysis II.
4. Ruslan A. Sharipov. Course of analytical geometry.
5. Jim Hefferon. Linear Algebra.
6. Ruslan A. Sharipov. Course of linear algebra and multidimensional geometry.
7. Gilbert Strang. Linear algebra and its applications.
8. W. Keith Nicholson. Linear Algebra with Applications.
9. William E. Boyce, Richard C. DiPrima. Elementary Differential Equations and boundary value problems.
10. Dmitri P. Bertsekas, John N. Tsitsiklis. Introduction to Probability, 2nd Edition.
11. Joseph K. Blitzstein, Jessica Hwang. Introduction to Probability.
12. G. Cain. Complex analysis.
13. T. Gamelin. Complex analysis.
14. Yehuda Pinchover, Jacob Rubinstein. An introduction to partial differential equations.