

## **PROGRAM OF ENTRANCE TEST IN MATHEMATICS FOR STUDENTS ENTERING MASTER PROGRAM**

The entrance test is a written exam, consisting of 6-10 problems/tasks of different difficulty levels. All problems require a complete solution with a detailed proof/explanation. The following resources are allowed during the exam:

- Wikipedia.org;
- Wolframalpha.com;
- Live.sympy.org;
- Python.org/shell;
- CoCalc (SageMath).

Counting an answer using one of these systems is not a complete solution or a proof. These resources can only be used as a hint.

To complete the tasks given 3 astronomical hours.

### **Foundations of algebra**

1. Groups, Abelian groups, normal subgroups, classical examples: groups of numbers by addition and multiplication, a group of non-degenerate matrices, a group of permutations, a group of residues by addition and multiplication.
2. Rings and commutative associative rings with unit. Examples: rings of numbers, rings of matrices, rings of residues, rings of polynomials.
3. Fields, definition and examples: the field of rational numbers, real numbers, complex numbers.

### **Basics of calculus**

1. Sequences. Limits of sequences. Examples of convergent and divergent sequences.
2. Continuous functions of one variable. Limits of functions.
3. Derivative. Differentiable functions. Mean value theorems: Fermat, Roll, Lagrange, Cauchy.
4. Infinitely small and limited quantities. Big-O notation.
5. Taylor series.
6. Indefinite integrals. Antiderivative.
7. Definite integrals. Improper integrals

### **Literature for self-study**

1. E.B. Vinberg, «A Course in Algebra», Graduate Studies in Mathematics, AMS, Vol. 56, 2003.
2. V.A. Zorich, «Mathematical Analysis I», Springer-Verlag Berlin Heidelberg, 2004.
3. L.B. Korolov, Ya.G. Sinai, «Theory of Probability and Random Processes», Springer-Verlag Berlin Heidelberg, 2007.
4. W. Rudin, «Principles of Mathematical Analysis», International Series in Pure and Applied Mathematics, McGraw-Hill Education, 1976, 3<sup>rd</sup> Edition.
5. R. Stanley, «Enumerative Combinatorics», Cambridge Studies in Advanced Mathematics, Cambridge University Press, 2011, 2<sup>nd</sup> Edition.