

# PROGRAM OF ENTRANCE TEST IN SPECIALTY FOR STUDENTS ENTERING MASTER PROGRAM

The entrance test is a written exam, consisting of 6-10 problems/tasks of different difficulty levels. All problems require a complete solution with a detailed proof/explanation. The following resources are allowed during the exam:

- Wikipedia.org;
- Wolframalpha.com;
- Live.sympy.org;
- Python.org/shell;
- CoCalc (SageMath).

Counting an answer using one of these systems is not a complete solution or a proof. These resources can only be used as a hint.

To complete the tasks given 3 astronomical hours.

## **Elements of linear algebra**

1. Systems of linear equations and the Gaussian elimination.
2. Vector spaces. Definition, examples: a space of rows, spaces of square matrices, spaces of symmetric and skew-symmetric square matrices, spaces of polynomials of one variable.
3. Linearly independent and linearly dependent systems of vectors.
4. A basis and the dimension of a vector space.

## **Foundations of topology in $R^n$**

1. The topology of the real line. Intervals and segments. Convergent subsegments. Open and closed sets.
2. Open and closed sets in a multidimensional space.
3. Continuous maps.
4. Compact subsets in  $R^n$ : finite subcovers. Closure and boundedness.

## **Combinatorics and probability**

1. Pigeonhole (Dirichlet's box) principle (with 2-3 examples of its application).
2. Standard counting rules: the rule of sum and the rule of product.
3. Combinations, placements and permutations.
4. Newton's binomial theorem.
5. Elementary events and finite sample spaces. The classical definition of probability. Computation of probabilities in classical settings.
6. Graphs: definitions. Complete graphs, simple graphs, trees, cycles. Degrees of vertices.

## **Literature for self-study**

1. E.B. Vinberg, «A Course in Algebra», Graduate Studies in Mathematics, AMS, Vol. 56, 2003.
2. V.A. Zorich, «Mathematical Analysis I», Springer-Verlag Berlin Heidelberg, 2004.
3. L.B. Korolov, Ya.G. Sinai, «Theory of Probability and Random Processes», Springer-Verlag Berlin Heidelberg, 2007.

4. W. Rudin, «Principles of Mathematical Analysis», International Series in Pure and Applied Mathematics, McGraw-Hill Education, 1976, 3<sup>rd</sup> Edition.
5. R. Stanley, «Enumerative Combinatorics», Cambridge Studies in Advanced Mathematics, Cambridge University Press, 2011, 2<sup>nd</sup> Edition.