MOSCOW INSTITUTE OF PHYSICS AND TECHNOLOGY

ENTRANCE EXAMINATION ON MATHEMATICS

PROBLEM SET 3 CODE ____________

Filled in be the executive secretary

Part 1

In this part of the examination you only need to provide answers to all the problems, and you do not need to submit your solutions. The answer is either an integer, or a finite decimal fraction, e.g. -37 or $2{,}208$.

1. Points M and N belong to lateral sides AB and CD of trapezoid $ABCD$, and $AM : MB = DN : NC = 2 : 5$. It is known that $BC = 16$, $MN = 11$. Find AD.
2. The population of a town was equal to $30\,000$. After a few years the population increased by $\alpha\%$, and after several more years it increased by $\alpha\%$ once more. As the result, the population became equal to $33\,075$. Find α.
3. Solve the equation $x^3 - 3x^2 - 6x + 8 = 0$. As the answer, indicate its largest root. If it does not exist write down 2021.
4. Find the value of the expression $\dfrac{\frac{1}{\sqrt{2}} + \sqrt{2} + \sqrt{3} - \frac{1}{\sqrt{3}}}{\sqrt{1{,}5} + 1} \cdot \dfrac{15 + 3\sqrt{6}}{\sqrt{3}}$.
5. Find the smallest root of the equation $\sqrt{2x+3} - \sqrt{4-x} = \sqrt{7-x}$. If there is no smallest root, indicate 2021 as the answer.
6. Two trains depart simultaneously from cities A and B, the first one moving from A to B, and the second one from B to A. They meet in 30 hours, and the first train arrives to B 25 hours later than the second train arrives to A. Find the time the slowest train needs for all the journey. Express your answer in hours.
7. α and β are acute angles of a right triangle, and they satisfy the relation $\sin 2\alpha = 1 + \sin(3\alpha - \beta)$. Find the largest possible value of β. Express your answer in degrees.

Part 2

In this part you need to submit solutions to all the problems. ***The problems without solutions are not graded.***

8. Find the sum of all natural numbers that do not exceed $1\,280$ and that are not divisible by 17.
9. Point A belongs to side QR of triangle PQR. It is known that $\angle APQ = \angle APR = \frac{\pi}{6}$; $AP = 10\sqrt{3}$, $AQ : AR = 2 : 5$. Find the area of the triangle.
10. Solve the equation $2\log_9^2 x = \log_3 x \cdot \log_3\left(\sqrt{2x+1} - 1\right)$.

Part 3

In this part you need to submit solutions to all the problems. ***The problems without solutions are not graded.***

11. A square is inscribed into an isosceles triangle in such a way that two of its vertices belong to the base of the triangle, and two other vertices lie on its lateral sides. Find the side of the square if the sides of the triangle are equal to 25, 25, 30.

12. Solve the system of equations $\begin{cases} \frac{2}{x^2+3xy} + \frac{3}{y^2-xy} = \frac{25}{14}, \\ \frac{3}{x^2+3xy} - \frac{2}{y^2-xy} = -\frac{4}{7}. \end{cases}$

13. Find all values of parameter a such that equation $(a+4x-x^2-1)(a+1-|x-2|)=0$ has exactly 3 real roots.